Progress Report

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Recall the Stein variational descent algorithm:

1. Choose a target density f, and a collection of points $\{x_i^0\}_{i=1}^n$.
2. Let $\hat{\phi}_\ell^*(x) = \frac{1}{n}\sum_{j=1}^n k(x_j^\ell, x)\nabla_{x_j^\ell} \log f(x_j^\ell) + \nabla_{x_j^\ell} k(x_j^\ell, x)$.
3. Define recursively $x_i^{\ell+1} = x_i^\ell + \varepsilon_\ell \hat{\phi}_\ell^*(x_i^\ell)$.

Let L_k be a function such that given $X = (x_1, \ldots x_n)$ N IID samples from a distribution with PDF f, $L_k(X) \approx (\log f)'(x_k)$.

In my research, I study dynamical systems defined by the Hamiltonian

$$H = \sum \frac{1}{2} p_k^2 + \sum (\log f)'(x_k)^2 + \sum L_k(X)$$

Thinking about this as a discrete time-step approximation of an interacting particle system, $\phi^*(x_k)$ is the momentum of x_k. To relate this to a Hamiltonian dynamical system, we can recall that

$$\frac{dp_k}{dt} = -\frac{\partial H}{\partial x_k}.$$

Treating $x_k(t)$ as a continuous function of t, we will take its derivative. Note that since ϕ is a function of all of x_1, $\ldots$, x_n, we can't take

$$\frac{d\phi(x_k(t))}{dt} = \phi'(x_k)x_k'(t) = \phi'(x_k)\phi(x_k).$$

The correct derivative

$$\frac{d\phi(x_k(t))}{dt} = \frac{1}{n}\sum \frac{d}{dt}\left[k(x_j, x_k)\right](\log f)'(x_j) + k(x_j, x_k)(\log f)''(x_j)\phi(x_j) + \frac{d}{dt}\partial_1 k(x_j, x_k)$$

Taking the partial derivative of H,

$$\frac{\partial H}{\partial x_k} = 2(\log f)'(x_k)(\log f)''(x_k) + \sum \frac{\partial}{\partial x_k} L_j(X).$$

If I could find k, L so that $\frac{d\phi(x_k(t))}{dt} = \frac{\partial H}{\partial x_k}$, I'd be able to apply the results of my research to this, and vice versa.

Rate of convergence ($1/N$) for an infinite particle system are given in *A Non-Asymptotic Analysis for Stein Variational Gradient Descent* [Korba et al], under several assumptions. The paper also discusses finite particle systems, and gives a reason why this would be difficult to find.

References i

Qiang Liu and Dilin Wang. “Stein Variational Gradient Descent: A General Purpose Bayesian Inference Algorithm”. In: *Advances in Neural Information Processing Systems*. Ed. by D. Lee et al. Vol. 29. Curran Associates, Inc., 2016. URL: https://proceedings.neurips.cc/paper/2016/file/b3ba8f1bee1238a2f37603d90b58898d-Paper.pdf.