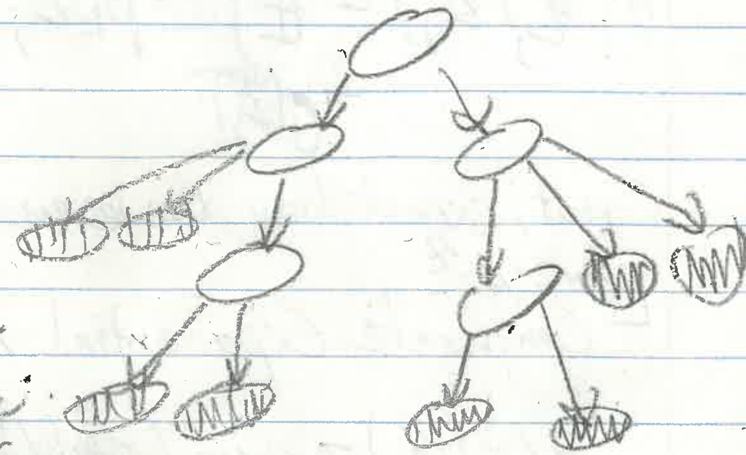
CSC 665

1/

9/16/19

Variational Message Passing *

Message passing
- Sum-product
- Max Sum



* Key Limitation - tree structure for exact inference

Variational Inference

Find ~~$\max_q -KL(p(\theta|x) || q(\theta))$~~

This is wrong

$D_{KL}(Q||P) = \sum_Z Q(Z)\left[\log \frac{Q(Z)}{P(Z,X)} + \log P(X)\right]$

intract-able

$\ln q_j^*(Z_j) = \mathbb{E}_{\neg q_j(Z_j)}\left[\ln p(X,Z)\right] + \text{const}$

Mean-field
Belief prop.
Expect. Prop
(See notes)

* Key limitation (Mean-field): Q^* (local max) marginally independent

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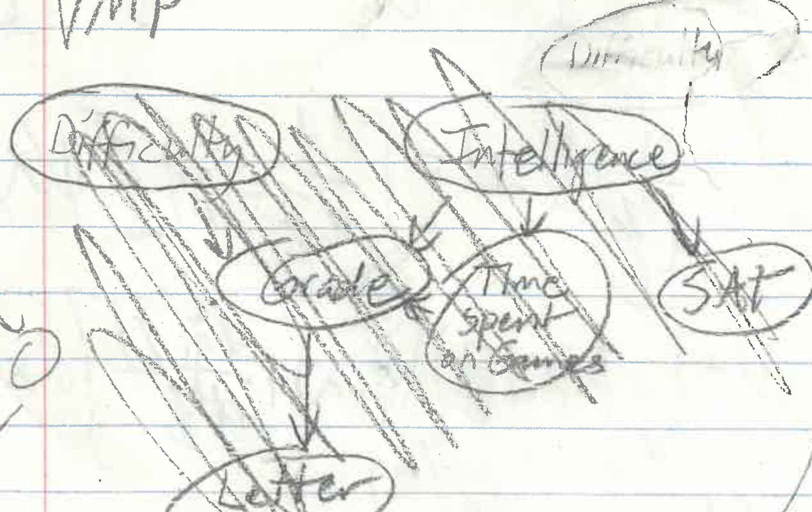
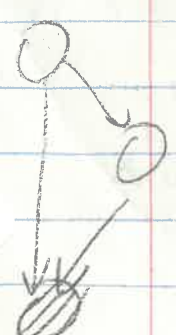
VMP



Markov Blanket (of "Grade")

Return to *
(And show fig 1)



* Decompose using graph structure

$$\ln q_j^*(Z_j) = \mathbb{E}_{\neg q(Z_j)}\left[\ln p(Z_j \mid pa_j)\right] + \sum_{k \in ch_j} \mathbb{E}_{\neg q(Z_j)}\left[\ln p(Z_k \mid pa_k)\right] + \text{const}$$

Great, except how do we ensure a tractable distribution for q_j?

Conjugate Exponential Models

$$P(X \mid Y) = \exp\left[\eta(Y)^T \phi(X) + f(X) + \underbrace{g(Y)}_{\text{normalizer}}\right]$$

Calculate Expectation Goal:

$$\int \phi(x) P(X \mid Y) = \text{something tractable}$$

Getting There:

$$P(X \mid \eta) = \exp\left[\eta^T \phi(X) + f(X) + \tilde{g}(\eta)\right]$$

$$\int_X P(X \mid \eta)\, dX = 1 = \int \exp\left[\eta^T \phi(X) + f(X) + \tilde{g}(\eta)\right] dX$$

$$\frac{d}{d\eta}(1) = 0 = \int_X \frac{d}{d\eta} \exp\left[\eta^T \phi(X) + f(X) + \tilde{g}(\eta)\right] dX$$

$$0 = \int_X P(X \mid \eta)\left[\phi(X) + \frac{d\tilde{g}(\eta)}{d\eta}\right] dx$$

$$\Rightarrow \mathbb{E}_{P(X \mid \eta)}\left[\phi(X)\right] = -\frac{d\tilde{g}(\eta)}{d\eta}$$

CSC 665 Notation: Y is parent node

X is child node

Parent Message!

$$m_{Y\to X} = \mathbb{E}_{q(X|Y)}\left[\Phi(X)\right] = \frac{d\tilde{g}(\eta)}{d\eta}$$

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Child message? $\mathbb{E}\left[\ln P(X|Y, cp_Y)\right]$

$$\ln P(X|Y, cp_Y) = \underbrace{\eta_{XY}(X, cp_Y)^T \Phi_Y(Y) + \lambda(X, cp_Y)}_{\text{Why?}}$$

Conjugacy!

In total, opt. step is: (p. 668)

$\ln q^*_Y(Y) =$

eqn 16 is key! Defines the two messages